GRADE XII PHYSICS

ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

Electric Potential at a point in the electric field is defined as the work done in moving (without any acceleration) a unit positive charge from infinity to that point against the electrostatic force irrespective of the path followed.

If W is the work done in moving Q charge from infinity to a point, then the potential at that point is

Potential =work/charge

OR

$$V=W/Q$$

Electric Potential Difference between any two points in the electric field is defined asthe work done in moving (without any acceleration) a unit positive charge from one point to the other against the electrostatic force irrespective of the path followed.

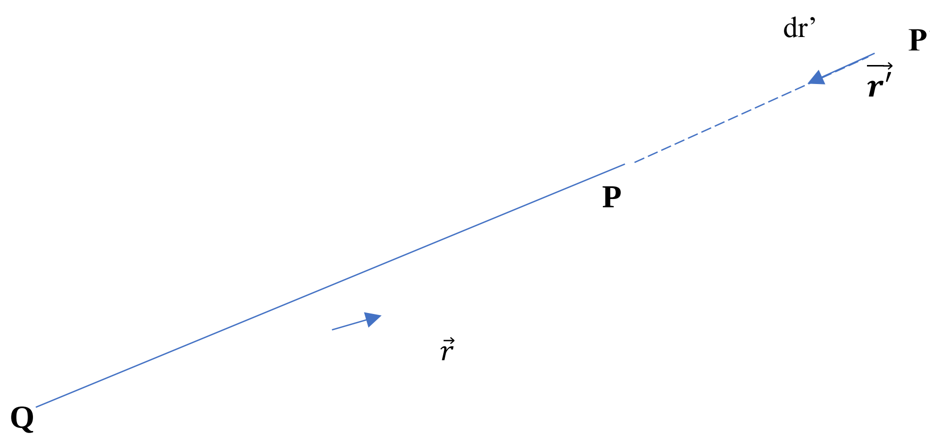
The S.I Unit of electric potential is volt(V)

One volt

Electric potential at a point is one volt if one joule of work is done in moving one coulomb charge from infinity to that point in the electric field.

POTENTIAL AT A POINT DUE TO A POINT CHARGE

Consider a point charge Q at the origin. (take Q to be positive). We wish to determine the potential at any point P with position vector r from the origin. For that we must calculate the work done in bringing a unit positive test charge from infinity to the point P. For Q > 0, the work done against the repulsive force on the test charge is positive. Since work done is independent of the path, we choose a convenient path – along the line joining the charge to the point P.



At some intermediate point P′ on the path, the electrostatic force on a unit positive charge is

$$\vec{F} = \frac{1}{4\pi \epsilon_0} \frac{Q}{r'^2} \widehat{r'}$$

where $\widehat{r'}$ is the unit vector along OP′.

Work done against this force from r′ to r′ + dr’ is

$$dw = \frac{-1}{4\pi \epsilon_0} \frac{Q dr'}{r'^2}$$

The negative sign appears because for $\Delta r' < 0$, ΔW is positive .
Total work done (W) by the external force is obtained by integrating Eq. (2.6) from r ′ = ∞ to r ′ = r,

$$W = \int_{\infty}^{r} dw$$

$$W = \int_{\infty}^{r} \frac{-1}{4\pi \epsilon_0} \frac{Q dr'}{r'^2}$$

$$= \frac{-Q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{r'^2} dr'$$

$$= \frac{-Q}{4\pi\epsilon_0} \left[\frac{-1}{r'}\right]_{\infty}^{r}$$

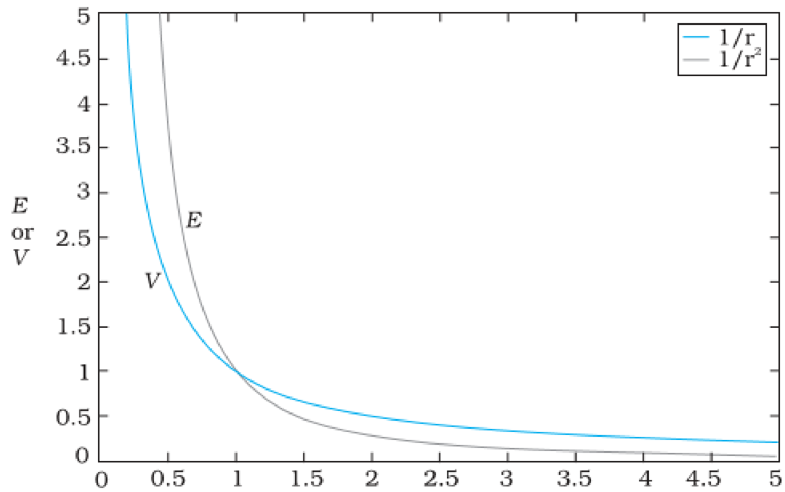
$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Therefore, the potential at P, $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

The potential difference between two points is independent of the path through which it is displaced. But depends only on the initial and final positions.

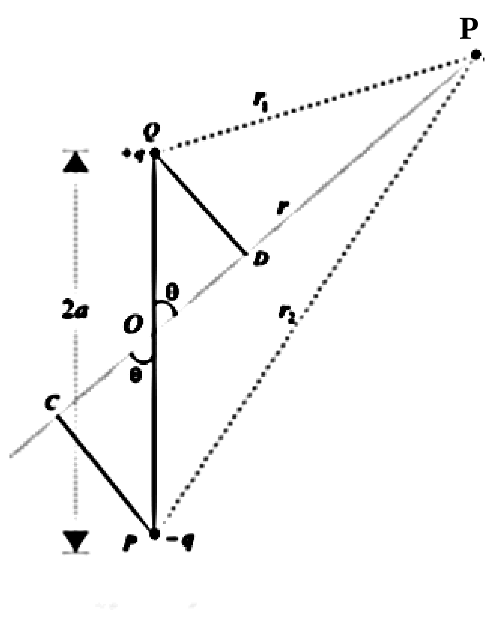
Electric field is a conservative field, because the work done in moving a charge from one point to another is independent of the path, but depends on ly on the initial and final positions, ie the work done by the electrostatic force over a closed path is zero.

The graph showing the variation of potential and field due to a point charge with distance.



POTENTIAL AT A POINT DUE TO AN ELECTRIC DIPOLE

Consider an electric dipole consisting of two-point charges +q and -q separated by a small distance 2a in air. Let us find the potential at appoint P which is at a distance r from the centre of the dipole O. O is considered as the origin.



Since electric potential obeys superposition principle so potential due to electric dipole as a whole would be sum of potential due to both the charges +q and -q. Thus

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

where r1 and r2 respectively are distance of charge +q and -q from point R.
Now draw line PC perpendicular to RO and line QD perpendicular to RO as shown in figure.
From triangle POC
cosθ= OC/OP=OC/a
therefore OC = a cosθ similarly
OD=acosθ

Now,
r_1 = QR≅RD = OR-OD
= r – a cosθ
r_2 = PR≅RC = OR+OC
= r +a cosθ
Therefore,

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$

since magnitude of dipole moment is

|p| = 2qa

$$\boldsymbol{V} = \frac{\mathbf{1}}{\mathbf{4\pi\epsilon_0}}\left(\frac{\boldsymbol{p\cos\theta}}{\boldsymbol{r^2 - a^2\cos^2\theta}}\right)$$

For a short dipole

If we consider the case where r>>a then, ie,

$$V = \frac{p\ cos\theta}{4\pi r^2}$$

Case 1. A point on the axial line.

For a point l on the axial line of the dipole, (θ = 0)

Therefore, cos θ = cos 0 = 1

Substituting this in the above expression

$$V = \frac{p}{4\pi \in_0 r^2}$$

Case 2. A point on the equatorial line.

For a point l on the axial line of the dipole, (θ = 90)

Therefore, cos θ = cos90 = 0

Substituting this in the above expression

V = 0

<u>Potential at a point due to two charges</u>

Consider two-point charges q_1 and q_2 kept in air at A and B respectively. The electric potential at a point P which is at distances r_1 and r_2 respectively from A and B respectively.

The potential at P due to the charge q_1, V_1 = $\frac{1}{4\pi\in_0}\frac{q_1}{r_1}$

The potential at P due to the charge q_2, V_2 = $\frac{1}{4\pi\in_0}\frac{q_2}{r_2}$

Therefore, the potential at P, V = V_1+ V_2 $= \frac{1}{4\pi\in_0}\frac{q_1}{r_1} + \frac{1}{4\pi\in_0}\frac{q_2}{r_2}$

$$V = \frac{1}{4\pi\in_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

Therefore potential at appoint due to n charges **V =** $\boldsymbol{\frac{1}{4\pi\in_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots + \frac{q_n}{r_n}\right]}$

Equipotential Surface

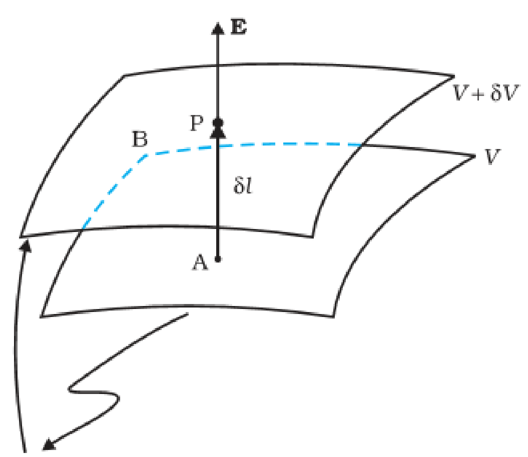
An equipotential surface is a surface in which the electric potential at all points on the surface will be the same.

Properties of equipotential surfaces

(1) Electric field lines are perpendicular to the equipotential surface
(2) Work done in moving a charge from one point to another point in an equipotential surface is zero.
(3) The equipotential surface due to a point charge is concentric spheres and that due to uniform electric field are planes normal to electric field.

General relation between potential difference and field

A and B are two equipotential surfaces separated by a small distance dl
Let the electric field at 'P' is E; then the work done in moving unit +ve charge from 'P' through a small distance dl against E is



$d\omega$ = E. di = -E dl

But, $d\omega$ = v+ dv –v = dv = potential difference.

∴ dv = - E dl

or E $= -\frac{dv}{dl}$

That is, the electric field is the negative gradient of potential

The negative sign shows that electric field is directed from higher potential to lower potential

ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF CHRGES
(In the absence of external electric field)

Potential energy of a system of charges is defined as the work done in bringing them from infinite separation to the present position.

Consider a system of 3 charges q_1, q_2 and q_3 in air. let the position vector charges are r_1, r_2 and r_3 respectively. Let us consider that these charges are kept initially at infinite separation. We can find the work done in bringing them from infinite separation to the present position.

The work done to bring q_1 from infinity to its present position is zero, because it is moved in a charge free region.

$$W_1 = 0$$

The work done to bring q2 from infinity to its present position is

$$W_2 = V_1 \times q_2$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

The work done in bringing the charge q_3 from infinity to 'r_3' is

$$W_3 = q_3 \, (v_1 + v_2)$$

$$= q_3 \left[\frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right]\right.$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

∴ **Total energy of the system is**

$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Potential energy a two charges in an external electric field.

Let us consider two charges q_1 and q_2 at two points with position vectors 'r_1' and 'r_2'. Let $V_{1(r1)}$ and $V(_{r2})$ be the electric potential at r_1 and r_2 due to the external electric field, then

∴ The work done in bringing q_1 from infinity to r_1 is

$$W_1 = q_1 V(r_1)$$

The work done in bringing q_2 from infinity to r_2 is

$$W_2 = q_2 V(r_2)$$

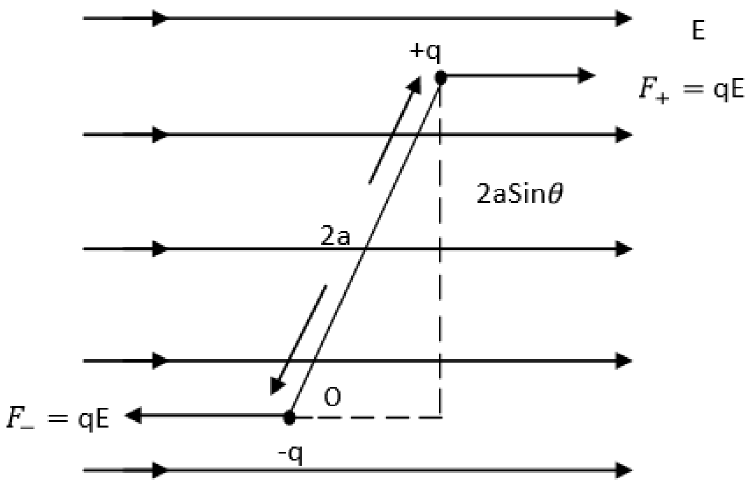
The work done in bringing the charge 'q_2' against the field of q_1 is

$$W_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

The total work done will be stored as the potential energy of the system as given by

$$\mathbf{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + q_1 V(r_1) + q_2 V(r_2)}$$

<u>Potential energy of a dipole in an external field</u>



$d\omega = \tau d\,\theta = p\,E\,Sin\theta\,d\theta$

The work done in rotating the dipole from an angle $\theta_1$to θ_2 is given by

$$W = \int_{\theta_1}^{\theta_2} pEsin\theta d\theta = -pE[scos\theta]_{\theta_2}^{\theta_1}$$

$$= -pE(cos\theta_2 - cos\theta_1)$$

$$W = p\,E\,(cos\theta_1 - cos\theta_2)$$

The potential energy of the dipole at an angle θ with electric field can be obtained by integrating the above equation from $\pi/_2$ to θ

i.e. $\mathcal{U} = \int_{\pi/_2}^{\theta} \mathrm{pESin}\theta \mathrm{d}\theta$

$= pE \int_{\pi/_2}^{\theta} \mathrm{Sin}\theta \mathrm{d}\theta$

$= pE\,[-Cos\,\theta]_{\pi/_2}^{\theta}$

$= -\,pE\,[Cos\,\theta - Cos^{\pi}/_2]$

$= -pE\,(Cos\,\theta - 0)$

U = -PE Cos θ or U =$-\vec{p}.\vec{E}$

Special cases:

U is max: if θ = 180^0, ie. U_{max} = PE

U is zero: if θ = 90^0, ie. U_{zero} = 0

***U* is minimum: if θ = 0^0, ie. U_{mm} = -PE**

Capacitor and Capacitance

A capacitor is a system consisting of two conductors separated by a an insulator. Let the charge on the conductor is Q and the potential difference across the m is V.

Then Q α V

Q = C V, where C is the constant of proportionality known as capacitance of the capacitor.

Therefore, C = $\frac{Q}{V}$.

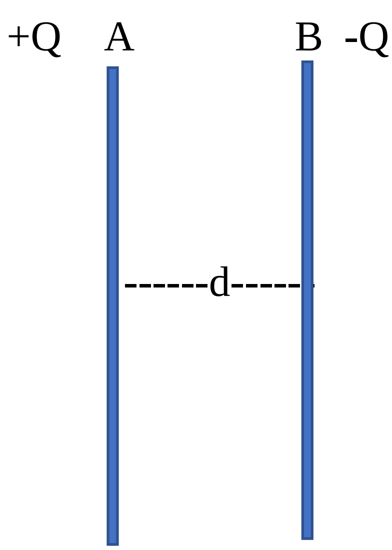
The S.I unit of capacitance is C/V but it is called farad(F)

The capacitance depends up on the shape and dimensions of the conductor and the nature of the medium in between them.

There are different types of capacitors like Parallel plate capacitor, spherical capacitor, ceramic capacitor, variable capacitor etc.

Capacitance of a parallel plate capacitor.

A Parallel Plate Capacitor is formed by two identical parallel conducting plate separated by a small distance of ‘d. Let A be the area of each plates Q is the charge in the plate and σ is the surface density of charge and d is the separation between the plates.



The electric field in the region between the plates of the capacitor is $E = \frac{\sigma}{\epsilon_0}$

The electric field between the plates of the capacitor is uniform.

The potential difference between the plates $V = E\,d$

Therefore the capacitance of the capacitor $C = \frac{Q}{V} = \frac{\sigma A}{E\,d} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0} d} = \frac{\epsilon_0 A}{d}$

$$\mathbf{C} = \frac{\epsilon_0 \boldsymbol{A}}{\boldsymbol{d}}$$

Dielectrics and polarization

Dielectrics are non-conducting polar or non-polar substances. If the centres of positive and negative charges of a molecule do not coincide, then it is a polar molecule and otherwise non polar molecule. In the polar molecule there will be a permanent dipole moment. But the net dipole moment of the substance becomes zero because of the random orientation of each molecule of the substance.

If a non-polar substance is placed in an external electric field, each molecule will change into dipoles in the direction of the external field and producing an induced field in the opposite direction.

If a polar substance is placed in an external electric field, each dipole molecule will align in the direction of the electric field and producing an internal field in the opposite direction.

But in both cases, the internal field produced will be less than the external field and there by the net field become

$$E = E_0 - E_p$$

The effect of dielectric in a capacitor

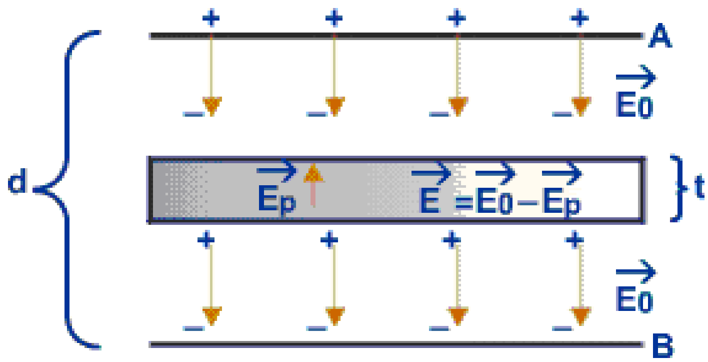
When a dielectric is placed in between the plates of a parallel plate capacitor due to electric polarization, the electric field and hence the potential between the plates is decreased. Hence the capacitance of the capacitor is increased by dielectric constant (K) times.

Let us consider a parallel plate capacitor of plate area ‘A’ and separation ‘d’. If the space between the plates is air, its capacitance is given by,

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Let $\pm Q$ be the charges and $\pm\sigma$ be the charge densities on the plate. Let and $V_0 = E_0\, d$ be the electric field and potential in this case.

When a dielectric slab is introduced between the plates of a capacitor an electric field is developed inside the dielectric slab in a direction opposite to the applied field. The electric field so developed is called polarization field **Ep**. Let **E0** be the applied field. The net electric field inside the dielectric slab **E= E0 - Ep**. Here electric field decreases, potential also decreases. So, capacitance increases.



The potential difference between the plates of the capacitor after introducing the dielectric slab of thickness t < d is given by

$$V = E.t + E_o.(d-t)$$

But $E = \frac{E_0}{K}$, where K is the dielectric constant of the medium between the plates.

$$\text{Therefore, } C = \frac{Q}{V} = \frac{\sigma\, A}{E.t - E_0(d-t)}$$

$$= \frac{\sigma A}{\frac{E_0}{k}t - E_{0(d-t)}}$$

$$= \frac{\sigma A}{E_{0\left\{\frac{t}{k}-(d-t)\right\}}}$$

But $E_0 = \frac{\sigma}{\epsilon_0}$

Therefore, $$\mathbf{C = \frac{\epsilon_0 A}{\frac{t}{k} - (d-t)}}$$

If the entire region between the plates is filled with dielectric, t = d

$$\boldsymbol{C = \frac{K\epsilon_0 A}{d}}$$

Energy stored in a capacitor

Let us consider a parallel plate capacitors of area A and separation of the plates as 'd' so that the capacitance is

$$C = \frac{\varepsilon_0 A}{d}$$

If a charge 'q' is given to the capacitor, its potential is 'v' so that

$$C = \frac{q}{v} \qquad \text{or} \qquad V = \frac{q}{C}$$

Now the work done in giving an additional charge dq to it is

$$d\omega = Vdq$$

$$\text{i.e. } d\omega = \frac{q}{v}\,dq$$

Now the total work done in charge the capacitor from q=0 to q=Q is obtained by integration as

$$\omega = \int_0^Q \frac{q}{v}\,dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$= \frac{1}{C}\frac{Q^2}{2}$$

$$= \frac{Q^2}{2C}$$

This work done is stored in the capacitor as electro static potential energy in the electric field between the plates.

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Energy density

If V = Ed and C = $\frac{\varepsilon_0 A}{d}$

Then, U = $\frac{1}{2}$ CV^2

$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 A E^2 d^2}{d}$$

$$= \frac{1}{2}\varepsilon_0 E^2 (Ad)$$

∴ The energy density or energy per unit volume of the capacitor is

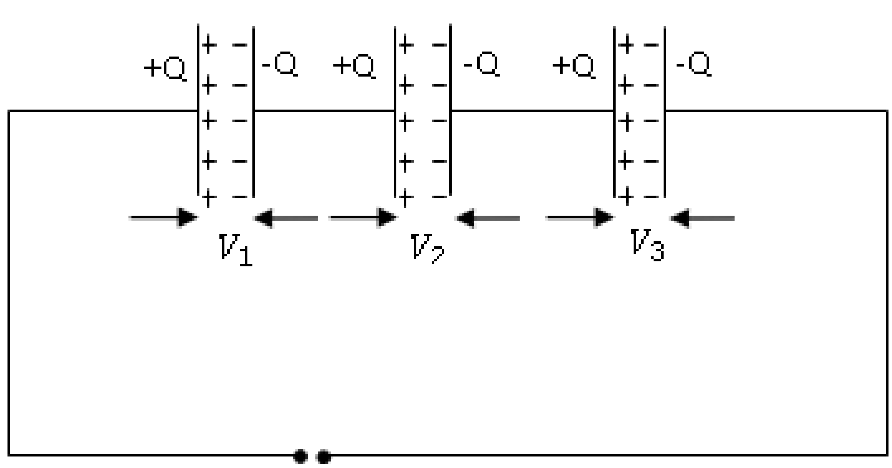
$$U_d = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Combination of capacitors

Capacitors can be connected together in a circuit in two different ways, series combination and parallel combination

(1) Series combination

Let us consider three capacitors C_1, C_2 and C_3 connected in series with a voltage source 'v' as shown below.



If the Capacitors are connected in Series the same charge ‘Q’ will be reaching to all the capacitors and the potential ‘V’ is divided into V_1, V_2 and V_3 across C_1, C_2 and C_3 respectively.

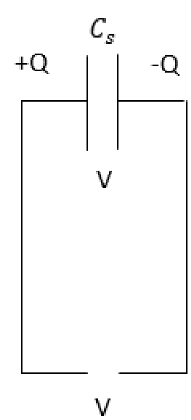
$$\therefore V = V_1 + V_2 + V_3 \longrightarrow (1)$$

But, $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$ and $V_3 = \frac{Q}{C_3}$

$$\therefore V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$= Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \longrightarrow (2)$$

If these capacitors are replaced by an equivalent capacitance ‘C_S’ such that the voltage ‘V’ and the charge ‘Q’ remains the same, then



$$V = \frac{Q}{C_s} \longrightarrow (3)$$

From Equation: (2) and (3)

$$\frac{Q}{C_s} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

i.e. $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If n capacitors are connected in series then

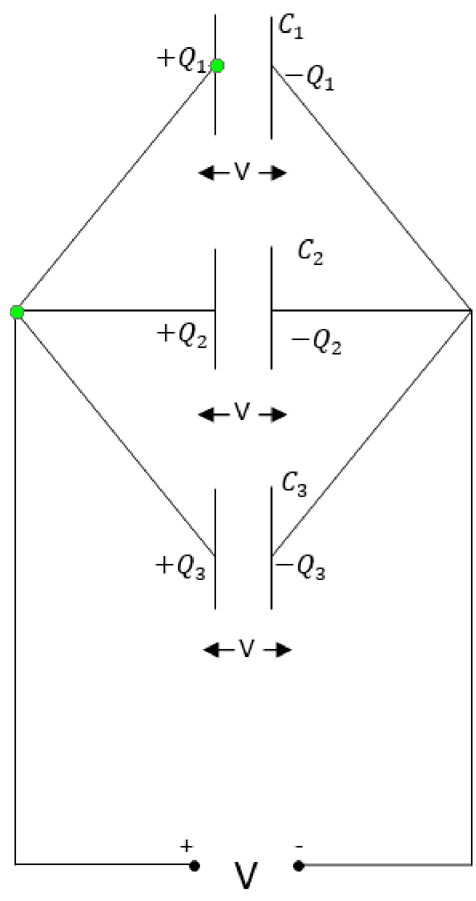
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots + \frac{1}{C_n}$$

i.e. The reciprocal of the effective capacitance in series combination is equal to the sum of the reciprocals of the individual capacitances.

If 'n' identical capacitors of C each are connected in series, then

$$C_s = \frac{C}{n}$$

(2) Parallel combination



If the capacitors are connected in parallel, the potential difference across each capacitor will be same and equal to the source voltage, but the charge is distributed as Q_1, Q_2 and Q_3 so that the charge from the source

$$Q = Q_1 + Q_2 + Q_3 \longrightarrow \quad (1)$$

But $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$

$$\therefore\ Q = C_1V + C_2V + C_3V$$

$$= (C_1 + C_2 + C_3)\,V \longrightarrow \quad (2)$$

If these three capacitors are replaced by a single capacitance 'Cp' in such a way that the voltage 'V' and the charge 'Q' remains the same, then

$$Q = C_pV \longrightarrow \quad (3)$$

From (2) and (3)

$$C_pV = (C_1 + C_2 + C_3)\,V$$

i.e. $\mathbf{Cp = C_1 + C_2 + C_3}$

If 'n' capacitors are connected in parallel then

$$C_p = C_1 + C_2 + C_3 + \ldots\ldots\ldots\ldots\ldots\ldots + C_n$$

i.e. The effective capacitance in parallel combination is equal to the sum of the individual capacitances.

If 'n' identical capacitors of each 'c' are connected in parallel, then

$$\mathbf{C_P = nc}$$